Economics of mixed-species forestry with ecosystem services*

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Abstract

Specifying forest value besides raw material production by the Faustmann-Hartman setup is widely established, but criticized as restrictive in capturing diversity values. We show that extending the model to cover diversity attributes, i.e. including mixed species and internal heterogeneity within species is not enough to overcome the restrictions. Additionally, it is necessary to extend forest harvesting regimes to cover thinning (partial harvesting), continuous cover forestry, and the management of commercially useless trees. Restrictions in the Faustmann-Hartman setup are first shown analytically with optimized thinning, but without tree size structures. The empirical significance of these findings is shown by a model with four tree species, tree size structures, an extended set of forest management activities, a detailed description of harvesting costs, and a measure for stand diversity as a key factor behind ecosystem services. We show how optimal harvesting regime, net revenues, wood output, and stand diversity depend on model flexibility, economic parameters and on the valuation of ecosystem services. In a setup allowing flexible management regimes, the costs of reaching a specified level of ecosystem services are negligible compared to the Faustmann-Hartman specification.

1 Introduction

Forests represent a prime example of extractive natural resources that are highly valuable besides their contribution as a source of raw materials. In resource economics, this was perhaps first formalized in Hartman (1976), who included the value of a standing forest into the generic Faustmann (1849) and Samuelson (1976) optimal rotation model. Since then, numerous studies have applied the Hartmann extension. However, despite this success, a recent review by Amacher (2015) comes to the conclusion that albeit the economic modeling of amenities as a function of stand age may be justified as a first hypothesis, it remains very basic. The uniformity of the large number of models presented after Hartmann (1976) is also found restrictive. According to Amacher (2015), forest economics has so far been unable to merge conservation biology, which identifies links between key habitats and species diversity attributes in economic optimization models of forest resources.

Our study aims to proceed from the Faustmann-Hartman setup by a model for mixed-species forests. This extension together with the inclusion of tree size structure enables us to formalize both species and intraspecies diversity. We show that extending the diversity attributes in the amenity or ecosystem services (ES) valuation is not enough as such. In addition, it is necessary to extend the set of forest management activities and proceed beyond Faustmann-Hartman rotation forestry regime based on harvesting stands by clearcuts only.[1]

Large literature suggests a strong connection between forest structural diversity, ES, and productivity. The 'habitat heterogeneity hypothesis' in ecology (MacArthur & MacArthur 1961) postulates that structurally complex habitats provide more niches and diverse ways of exploiting environmental resources and thus imply higher species diversity and productivity. Liang et al. (2016)

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[1] It is seldom noticed that in his famous formula for the bare land value, Faustmann (1849) includes the present value of revenues from thinning (partial cuttings).

study the biodiversity-productivity relationship using a global dataset, and find a positive concave relationship between species richness and tree volume productivity. Ongoing species loss in forests is found a threat to forest productivity, and benefits from the transition of monocultures to mixed-species stands are emphasized. Gamfelt et al. (2012) study boreal forests, and find either a monotonically increasing positive, or a single-peaked relationship between tree species richness and the supply of ES such as soil carbon storage, deadwood, and berry and game production. Zhou (2017) applies hedonic valuation and finds stand structural diversity and the density of large pine trees as key determinants for preserving amenities among forest owners in the U.S. southern pine region.

Another line of research has studied the connection between management measures and forest diversity characteristics. Bose et al. (2013) review expanding Canadian experiments on forests thinning and continuous cover forestry that aim to balance economic and ecological objectives. Avoiding clearcuts contributes to maintaining the natural characteristics of forest landscapes and more favorable habitat attributes for birds, insects, vertebrates, and vegetation. As constraints to continuous cover forestry, they mention the threat of short-sighted high-grading, i.e. "harvest the best, leave the rest". Conceptual and simulation models for synthesizing empirical knowledge are proposed as the most promising way ahead. In a review on Canadian field experiments Ruel et al. (2013) find that many attributes of old-growth forests can be preserved with thinning, and diversity indices for partially harvested stands remain similar or very close to uncut forests. Given an old-growth forest as the initial state, they obtained a result that avoiding clearcuts causes a short-term decrease in profitability, while the long-term outcome may be the reverse (no optimization applied). Martin et al. (2018) emphasize less intensive management treatments as a main method for maintaining the diversity of Canadian boreal forests, but expect that economic viability will restrict broader developments of the alternatives for clearcutting.

A study by Duncker et al. (2012), based on a detailed mixed-species model for southwest Germany, found mixed-species stands based on continuous cover forestry more favorable for balancing

wood production and ES than single-species rotation forests. For Nordic boreal forests similar results have been obtained in Peura et al. (2018).

Forest ecological and management research supports the view that while longer rotation periods are favorable for amenities and ES, the role of lower impact harvesting methods, such as thinning and continuous cover management, may be more important when the aim is to maintain forest diversity, ES, and a balance with wood production. This background strongly suggests extending the economic analysis of harvesting methods[2] and diversity attributes beyond the Faustmann-Hartman setup.

We first develop analytical results by a model with thinning in Clark (1976, p. 62) extended to include ES, multiple tree species, and continuous cover forestry. This model reveals the highly restrictive nature of the Faustmann-Hartman setup compared to a model with a wider set of harvesting options, and supports studying the significance of the theoretical findings by an empirically detailed model.

Our empirical extension is a detailed size-structured model[3] for stand growth and any number of tree species. The detailed structure enables describing stand diversity with the available ecological measures. The model allows three different types of harvesting activities: clearcut, thinning, and felling (noncommercial) trees without hauling from the site and optimization between rotation and continuous cover forestry. Revenues are separate for sawlogs and pulpwood, and market prices are species-specific. A detailed harvesting cost model recognizes various tree species and is separate for clearcuts, and thinning and felling trees without commercial value. Each harvest operation includes a fixed cost (transporting the harvester to the site). Thus, we optimize a vector of binary variables determining whether to harvest at the given period in addition to optimizing the harvested number of trees over the size classes and species. Natural regeneration implies that it is possible to continue thinning instead of

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[2] Nordic forest management practices typically include 2-3 thinnings before the clearcut but unlike rotation, thinnings are not optimized. Management based solely on thinning, i.e. continuous cover forestry has not been favored by forest experts due to the belief that it does not maximize sustainable yield.
[3] The size-structured model is investigated analytically in Tahvonen (2015) but for one species and without ES.

clearcutting followed by costly artificial regeneration. Excluding ES values and their dependence on diversity, all features and model parameters are based on empirical data valid in the Nordic context.

If thinning is ruled out, the number of tree species and intraspecies diversity has negligible effect on optimal rotation. Diversity valuation lengthens rotation and with very high valuation, yields an infinitely long rotation, i.e. abandon of forestry. This is strictly in line with the Faustmann-Hartman setup.

Adding the possibility of thinning more than doubles rotation length, increases bare land value by 20–70%, and yields higher stand diversity compared to rotation forestry, but lower levels of wood output. Even without ES preferences thinning (or partial cutting) allows to influence the development of tree and size structures in a way that is not possible within the Faustmann-Hartman model. Including the preferences for ES into the objective shows that a much lower level of their valuation is enough to abandon cleacuts compared to similar outcome within the Faustmann-Hartman model. ES preferences support continuous cover forestry, increases the share of birch, pine, and noncommercial broadleaves, large tree size classes and lengthens the continuous cover steady-state harvesting interval. Including thinning into the model cuts the cost of increasing ES production to a negligible level compared to the Faustmann-Hartman specification. Stand diversity also appears to be higher in the continuous cover steady state compared to an unharvested stand, even without including ES into the model objective.

Similar previous results in forest economics do not exist, and our results strongly suggest that proceeding beyond the Faustmann-Hartman setup is necessary to better understand the economics of forestry with preferences to ecosystem services and biodiversity. In existing literature, Buongiorno et al. (1994) include a diversity index into a model for continuous cover forestry, but use a static one species model. Lin et al. (1996) include mixed species, but no optimization. Getz and Haight (1989) optimize the management of California mixed forests, but without ES. Wikström and Erikson (2000) specify a one-rotation model for a two species forest, measure diversity with the Shannon index, but concentrate on the technical solution procedure. The background model in the hedonic price analysis of Zhou (2017)

describes continuous cover forestry, stand diversity is measured with the Shannon index, but no forest management details are analyzed.

We next specify the analytical model and results. Then we proceed to empirical model specifications and optimization methods. The results are first presented for the Faustmann-Hartman specification and then for the extension. Finally, we offer some conclusions.

2. Ecosystem services and thinning in mixed species model without size structure

Let $x_j(t)$, $j = 1,...,n$ denote the biomass volume of tree species j per land unit and $h_j(t)$, $j = 1,...,n$ the rate of thinning (harvesting) respectively. Denote $\mathbf{x} = [x_1, x_2,....x_n]$. Wood growth per tree species is given as $g_j(t) f_j(\mathbf{x})$, where g_j are aging functions and f_j growth functions with biological density dependence. ES values are $y(t) A(\mathbf{x})$, where function y denotes the dependence on stand age and function A the dependence on species volumes. Letting ρ denote the rate of interest, w the cost of artificial regeneration, p_j, $j = 1,...,n$ the stumpage prices and assuming quasi-linear preferences as in Hartman (1976) the model with thinning, clearcuts and ES valuation becomes

$$\max_{\{h_j, x_j(T), T \in [0,\infty), j=1,...,n\}} J = \frac{-w + \int_0^T \left[\sum_{j=1}^n p_j h_j + y(t) A(\mathbf{x}) \right] e^{-\rho t} dt + \sum_{j=1}^n p_j x_j(T) e^{-\rho T}}{1 - e^{-\rho T}} \tag{1}$$

$$s.t. \ \dot{x}_j = g_j(t) f_j(\mathbf{x}) - h_j, x_j(0) = x_{j0}, j = 1,...,n, \tag{2}$$

$$0 \le h_j \le h_{jmax}, j = 1,...,n. \tag{3}$$

Our inclusion of any number of species and ES values extends Clark (1976, p. 63) and Tahvonen (2016). Compared to the plantation forestry in Clark (1976, p. 63-) optimal rotation may be finite or infinite, i.e. the model includes optimal choice between continuous cover and rotation forestry. The ES valuation and growth functions are continuous and continuously differentiable and satisfy

$$y(t) > 0, \tag{A1}$$

$$A_{x_j} > 0 \text{ for } \sum_{j=1}^{n} \gamma_j x_j < \tilde{x}_j, \ A_{x_j} < 0 \text{ for } \sum_{j=1}^{n} \gamma_j x_j > \tilde{x}_j, \ \gamma_j \geq 0, \ A_{x_j x_j} < 0, \ j = 1,...,n, \tag{A2}$$

$$g_j(0) > 0, g_j' < 0, g_j'' > 0, g_j' \to \bar{g}_j \geq 0 \ \text{when} \ t \to \infty, j = 1,...,n \tag{A3}$$

$$f_{j x_j x_j} < 0, f_{j x_i} < 0, j = 1,...,n, \ i = 1,...,n, \ i \neq j \text{ and for each } \sum_{\substack{i=1 \\ i \neq j}}^{n} \mu_i x_i \text{ there exists } \hat{x}_j \geq 0 \text{ such}$$

$$\text{that} f_{j x_j} > 0 \text{ for } x_j < \hat{x}_j \text{ and } f_{j x_j} < 0 \text{ for } x_j > \hat{x}_j, \text{where } f_{j x_j}\left(\hat{x}_j, \sum_{\substack{i=1 \\ i \neq j}}^{n} \mu_i x_i \right) = 0 \text{ and } \mu_i \geq 0, i = 1,...,n. \tag{A4}$$

Additionally, functions A and $f_j, j = 1,...,n$ are assumed to be concave. The ES valuation function A

increases in the volume of each species, but only if total volume is not too high (A2). The growth of each

species decrease due to aging (A3), but may remain positive if natural regeneration occurs $\left(\bar{g}_j > 0 \right)$ (cf.

Clark 1976, p. 63- where $\bar{g}_j = 0$). Growth is density dependent and a high enough total stand density

implies that growth per species decreases (A4). Examples of these functions are

$$A(\mathbf{x}_t) = \sum_{j=1}^{n} a_j x_j - \eta_0 \left(\sum_{j=1}^{n} \eta_j x_j \right)^{\tau}, \ a_j > 0, \eta_0, \ \eta_j > 0, j = 0,...,n, \ \tau > 1,$$

$$f_j(\mathbf{x}_t) = x_j r_j \left(1 - \frac{x_j}{K_j - \sum_{\substack{i=1 \\ i \neq j}}^{n} \mu_i x_i} \right), \ r_j > 0, \ K_j > 0, \ \mu_i \geq 0, i = 1,...,n. [4]$$

Problem (1)–(3) can be solved by first optimizing thinning and assuming a fixed rotation period, and

next optimizing the rotation period given optimized thinning. Thus, for any rotation period $T > 0$ and

costate variables $\phi_j, j = 1,...,n$, the Hamiltonian and necessary optimality conditions are (2), (3) and

$$H = \sum_{j=1}^{n} p_j h_j + y(t) A(\mathbf{x}_t) + \sum_{j=1}^{n} \phi_j \left[g_j(t) f_j(\mathbf{x}_t) - h_j \right],$$

$$p_j - \phi_j < 0 \Rightarrow h_j = 0, \ j = 1,...,n, \tag{4}$$

$$p_j - \phi_j = 0 \Rightarrow 0 \leq h_j \leq h_{jmax}, \ j = 1,...,n, \tag{5}$$

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[4] Note that the denominator in f_j remains strictly positive when the initial levels of $x_{j0}, j = 1,...,n$ are low. The specification is close to that in Beckage and Gross (2006) but is concave in the admissible region.

$$p_j - \phi_j > 0 \Rightarrow h_j = h_{jmax}, \ j = 1,...,n, \tag{6}$$

$$\dot{\phi}_j = -A_{x_j} + \phi_j \left[\rho - g_j f_{jx_j} \right] - \sum_{\substack{i=1 \\ i \neq j}}^{n} \phi_i g_i f_{ix_j}, \ j = 1,...,n, \tag{7}$$

$$\phi_j - p_j \geq 0, x_j(T) \geq 0, \left(\phi_j - p_j \right) x_j(T) = 0, j = 1,...,n. \tag{8}$$

When $n = 1$, $A \equiv 0$, x_0 is low and rotation T is not too short, it is first optimal to let the stand grow without any thinning (regime $p - \phi < 0$, $h = 0$) until a regime switch to singular solution with $p = \phi$ and $\rho = gf_x$. Given $\rho < \bar{g}f_x(0)$, $\bar{g} > 0$, and conditions (8) this regime continues as interior solution until the end of any rotation $T \in (0, \infty)$, where the remaining $x(T)$ is clearut (Clark 1976, Tahvonen 2016).

Assuming ES values and any number of species we analyze a similar regime combination: wait, thin and clearcut at $T \in (0, \infty)$. Denote a solution satisfying the necessary optimal conditions (2), (3), (4)–(6), (7) and (8) as $h_j^*(t), 0 \leq t \leq T, x_j^*(T), j = 1,...,n$. Maximizing (1) w.r.t. T yields the condition for the solutions with finite optimal rotation denoted by $T*$

$$\frac{e^{-\rho T*}}{1 - e^{-\rho T*}} q(T*) = 0, \tag{9}$$

$$q(T*) \equiv \sum_{j=1}^{n} \bar{p}_j h_j^*(T*) + y(T*) A\left[\mathbf{x}(T*) \right] + \sum_{j=1}^{n} \bar{p}_j x_j^{*\prime}(T*) - \rho \left[\sum_{j=1}^{n} \bar{p}_j x_j^*(T*) + J(T*) \right] = 0. \tag{10}$$

At $T*$ the optimal regime is singular, i.e. $p_j - \phi_j = 0, \dot{\phi} = 0, j = 1,...,n$ implying that differentiating (10) and simplifying by (7) and $\dot{\phi}_j = 0, j = 1,...,n$ yields

$$q'(T*) = y'A + \sum_{j=1}^{n} p_j g_j' f_j. \tag{11}$$

In (11) the term $g' < 0$, but we have left the sign of y' unrestricted. Thus, without ES, optimal rotation

is unique but with ES valuation multiple locally optimal rotations can be ruled out only if

$y'(T^*)A(T^*)$ is negative or close enough to zero. Write equation (10) as

$$J_F(T) + J_A(T) = 0, \tag{12}$$

$$J_F(T) = \sum_{j=1}^n p_j x_j^{*'}(T^*) - \rho \sum_{j=1}^n p_j x_j^*(T^*) + \sum_{j=1}^n p_j h_j^*(T^*) - \rho \frac{-w + \int_0^T \sum_{j=1}^n p_j h_j e^{-\rho t} dt + \sum_{j=1}^n p_j x_j^*(T^*)}{1 - e^{-\rho t}} \tag{13}$$

$$J_A(T) = y(T^*)A[\mathbf{x}(T^*)] - \frac{\rho \int_0^{T^*} y(t)A[\mathbf{x}(t)]e^{-\rho t} dt}{1 - e^{-\rho T^*}}. \tag{14}$$

With no amenities the terms in (13)

$$\sum_{j=1}^n p_j x_j^{*'}(T^*) - \rho \sum_{j=1}^n p_j x_j^*(T^*) + \sum_{j=1}^n p_j h_j > 0 \tag{15}$$

by the concavity of f, $f_{i x_j} < 0, i \neq j$, conditions (7) and $\dot{\phi}_j = 0$. Given $\bar{g}_j > 0$ and $\rho < \overline{g} f_{j x_j}(0)$,

$j = 1,...,n$ the positive sign in (15) holds even when $t \to \infty$. Thus, given this case and a bare land value

(in 13) low enough for all $T \in [0, \infty)$, no finite rotation satisfying (9) exist implying that optimal rotation

is infinite, i.e. it is optimal to continue thinning without clearcut and continuous cover forestry is optimal.

Accordingly, if $\bar{g}_j = 0$, $j = 1,...,n$, the LHS of (13) is negative when $t \to \infty$, implying that there must be

at least one finite rotation satisfying (9). Thus, if growth cheeses independently on stand density as in

Clark (1976, p.63-), the optimal rotation is finite and clearcuts are inevitable.

ES values lengthen (shorten) optimal rotation if $J_A(T^*) > 0$ $(<)$ in (14). The former (latter) case

follows e.g. if $d(yA)/dt > 0$ $(<)$ for all $0 < t < T$. When $J_A(T) > 0$ as $t \to \infty$, ES values may cause a

regime switch from rotation forestry to continuous cover regime given rotation forestry happens to be

optimal without ES values. When $J_A(T) < 0$ as $t \to \infty$, ES values may cause a switch from continuous cover forestry to rotation forestry.
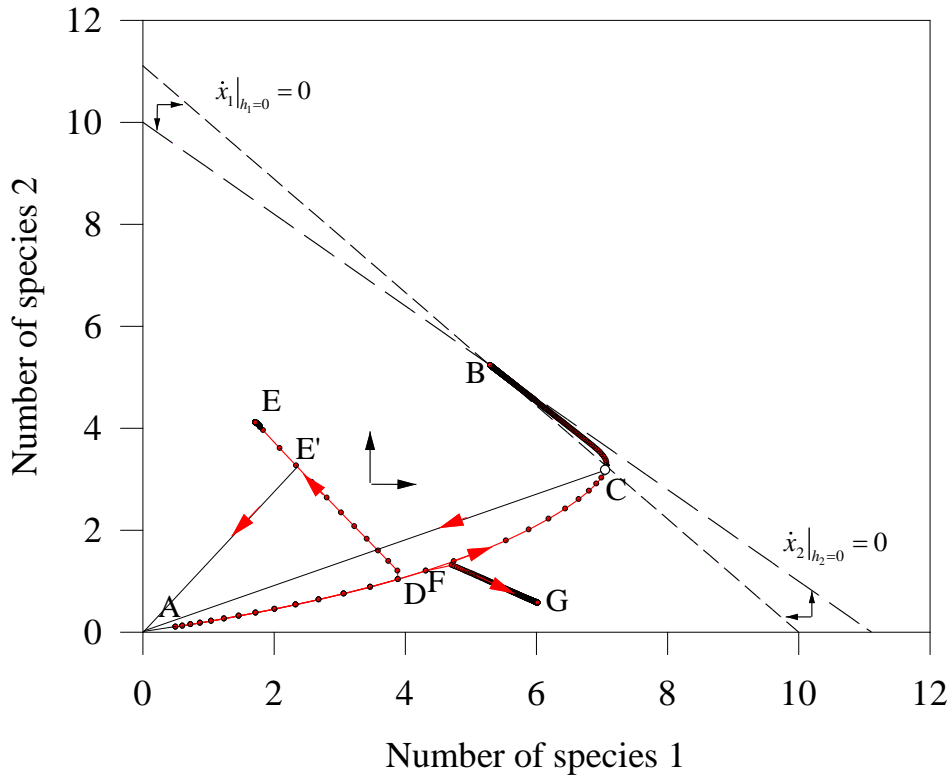

Figure 1. Comparison of admissible and optimal solutions with and without thinning

Note: $g(t)f(x) = \left(10 + 12e^{-0.02t}\right)x_j \, 0.01\left(1 - \dfrac{x_j}{K_j - 0.9\,x_i}\right)$, $j = 1,2, i = 1,2, \ j \neq i, x_{10} = 0.5, \ x_{20} = 0.1.$

Neglecting thinning and setting $h_j \equiv 0, j = 1,...,n$ in (13) implies that the solution collapses into the Faustmann-Hartman model for mixed stands. Compared to this, thinning expands both the set of stand development alternatives toward a possible clearcut or long run steady states. The fact that this makes a significant difference is shown for a two-species forest (Figure 1). When long run coexistence of the (unharvested) species is possible, an unharvested stand develops from an initial point A toward a stable steady state at B (via a point such as C). Within the Faustmann-Hartman setup, the choice set

includes stopping this development at any point between *A* and *B*, for example at point *C* and repeating the rotation from the origin or abandoning clearcut and approaching point *B*. After including thinning all such choices are available, but the admissible solution set additionally includes rotation cycles, such as $A, D, E', 0, A$, where path D, E' represents a singular solution with thinning. Point *B* is the only steady state within Faustmann-Hartman setup, while with thinning any non-negative point below both the isoclines represents admissible steady states with some thinning combination of the two species.

Assume the growth specification given in Figure 1 and

$$y(t)A(x_1, x_2) = 0.12\left(1 - e^{-0.06t}\right)\left[10x_1 + 20x_2 + 0.25\left(x_1 + x_2\right)^2\right], p_1 = 50, \ p_2 = 100, w = 400, \rho = 0.01. \tag{16}$$

The optimal solution for the Faustmann-Hartman model is a 27-periods rotation cycle $A, C, 0, A$. Without ES the rotation is 22 periods. With thinning and ES the optimal solution is the path *A,D, E*, i.e. a continuous cover solution, and without ES the solution is somewhat similar, but includes a clearcut at the age of 90 periods. Next assume the specification

$$y(t)A(x_1, x_2) = 0.13\left(1 - e^{-0.06t}\right)\left[40x_1 + 10x_2 + 0.25\left(x_1 + x_2\right)^2\right], p_1 = 100, \ p_2 = 50, w = 400, \rho = 0.02. \tag{17}$$

The Faustmann-Hartman solution is unchanged (cycle *A,C,0,A*), but the solution with thinning is *A,F,G*. Given specification (16), the Faustmann-Hartman objective values are 367 and 1054 with and without ES, while the objective values are much higher with thinning, i.e.1184 and 2092 respectively.

This theoretical model and analysis strongly suggest that the inclusion of intermediate cuttings or thinning may greatly alter forest management solutions and increase the economic objective values in a model with multiple tree species and ES values. We turn to examine to what extent these findings are realized in an empirically realistic model for mixed-species boreal forests.

3 A model for mixed species size-structured stands with thinning and ecosystem services

We apply a nonlinear size-structured model for mixed stands that allows direct application of empirically estimated ecological and economic models and parameters along with indices for biodiversity. For this end, let $\mathbf{x}_{jt} = (x_{j1t},...,x_{jnt})$ denote the number of trees of species $j = 1,...,l$ in size classes $1,...,n$ at the beginning of period t and $\mathbf{x}_t$ a matrix for the number of trees in different species and size classes respectively.[5] The fraction $\alpha_{js}(\mathbf{x}_t), j = 1,...,l, s = 1,...,n$ of size class s trees of species j move to size class $s+1$ at the end of each period and the fraction $\mu_{js}(\mathbf{x}_t)$ of trees dies. Natural regeneration is $\phi_j(\mathbf{x}_t), j = 1,...,l$. Given h_{jst} denotes (commercially) harvested and $k_{jst}, j = 1,...,l, s = 1,...,n, t = t_0,...,T$ felled trees (i.e. trees left on the site), the development of the mixed species stand can be written as

$$x_{j1,t+1} = \phi_j(\mathbf{x}_t) + \left[1 - \alpha_{j1}(\mathbf{x}_t) - \mu_{j1}(\mathbf{x}_t)\right]x_{j1t} - h_{j1t} - k_{j1t}, \quad j = 1,...,l, t = t_0,...,T, \tag{18}$$

$$x_{j,s+1,t+1} = \alpha_{js}(\mathbf{x}_t)x_{jst} + \left[1 - \alpha_{j,s+1}(\mathbf{x}_t) - \mu_{j,s+1}(\mathbf{x}_t)\right]x_{j,s+1,t} - h_{j,s+1,t} - k_{j,s+1,t}, \quad j = 1,...,l, s = 1,...,n-1, t = t_0,...,T, \tag{19}$$

$$x_{jst_0} \text{ given, } j = 1,...,l, s = 1,...,n. \tag{20}$$

Cuttings may not occur every period and are restricted by the constraints

$$h_{jst} = \delta_t h_{jst}, \quad j = 1,...,l, s = 1,...,n, t = t_1,...,T, \tag{21}$$

$$k_{jst} = \delta_t k_{jst}, \quad j = 1,...,l, s = 1,...,n, t = t_1,...,T, \tag{22}$$

where $\delta_t : Z \in [0,1], t = t_0, t_0 + 1,...$ are binaries and specify the periods with positive cuttings $(\delta_t = 1)$. The stand is established at $t = 0 \ (< t_0)$ with a fixed cost w. Gross harvesting revenues are $R(\mathbf{h}_t)$ and the variable harvesting costs for thinnings and clearcuts $C_{th}(\mathbf{h}_t, \mathbf{k}_t)$ and $C_{cl}(\mathbf{h}_t, \mathbf{k}_t)$ respectively. Wood value and harvesting costs will depend of species, tree size and the quantity of wood harvested.

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[5] The fact the in section 2 $x_j, j=1,...,n$, and $\mathbf{x}$ denoted volumes but here the number of trees should not cause any confusion.

Based on Gamfelt (2012) we postulate that a higher level of biological diversity implies a higher level of valuable (nontimber) ES. This is specified by a functional relationship between a biodiversity measure d_t and the value of nontimber ES given as $\tilde{A}\{E[d(\mathbf{x}_t)]\}$, where d measures biodiversity as a function of stand state $\mathbf{x}$, function E the dependence of ES on biodiversity, and function $\tilde{A}$ the willingness to pay for nontimber ES. Given C^f denotes fixed harvesting cost, b the per annum discount factor and Δ the period length, we write the objective functional as

$$\max_{\{\mathbf{h}_{it},\mathbf{k}_{it},\delta_t,i=1,...,l,T\in[t_0,\infty)\}} \frac{-w+\sum_{t=t_0}^{T-1}\left\{R(\mathbf{h}_t)-C_{th}(\mathbf{h}_t,\mathbf{k}_t)-\delta_t C^f+\tilde{A}\{E[d(\mathbf{x}_t)]\}\right\}b^{\Delta(t+1)}+\left\{R(\mathbf{h}_T)-C_{cl}(\mathbf{h}_T,\mathbf{k}_T)-\delta_T C^f+\tilde{A}\{E[d(\mathbf{x}_T)]\}\right\}b^{\Delta(T+1)}}{1-b^{\Delta(T+1)}}.(23)$$

When the rotation period $T\in[t_0,\infty)$ is finite the solution is a rotation forestry and when it is infinitely long it represents continuous cover forestry. The latter becomes possible when trees regenerate naturally and thinning can be continued without clearcuts. In specification (23) land is initially bare but this can be extended to optimize cuttings from any initial stand state (Tahvonen 2015). Notice that by removing thinning the specification falls back to optimizing the rotation length only similarly as model (1)-(3).

4 Empirical specifications and data

The equations for stand growth take the form

$$\phi_j(\mathbf{x})=\frac{\phi_{j1}\beta_i(\mathbf{x})^{\phi_{j2}}S^{\phi_{j3}}\left[\beta(\mathbf{x})+\phi_{i4}\right]^{\phi_{j5}}}{1+e^{-\left[\phi_{j6}+\phi_{j7}\beta(\mathbf{x})+\phi_{j8}SI+\phi_{j9}\beta_{ij}(\mathbf{x})\right]}},\ j=1,...,l,\tag{24}$$

$$\alpha_{sj}(\mathbf{x})=\frac{1}{\alpha_{j1}}\left[\alpha_{j2}+\alpha_{j3}d_s+\alpha_{j4}d_s^2+\alpha_{j5}d_s^3+\alpha_{j6}\beta_s(\mathbf{x})+\alpha_{j7}\beta(\mathbf{x})+\alpha_{j8}S+\alpha_{j9}L\right],\ j=1,...,l,s=1,...,n,\tag{25}$$

$$\mu_{sj}(\mathbf{x})=\left\{1+e^{-\left[\mu_{j1}+\mu_{j2}d_2+\mu_{j3}d_s^2+\mu_{ij}\beta(\mathbf{x})\right]}\right\}^{-1},\ j=1,..,l,\ s=1,...,n,\tag{26}$$

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where $d_s, s = 1,...,n$ is tree diameter (cm) at a height of 130 cm, S denotes site index, L latitude and β, and β_s stand basal area and basal area (m^2) for trees with diameter larger than in size class s, respectively (Bollandsås et al. 2008). The parameter values for ϕ_{ji}, μ_{ji} and $\alpha_{ji}, j = 1,...,l, i = 1,...,7$ and details for the site index and latitude are given in Appendix 1, Table 1.
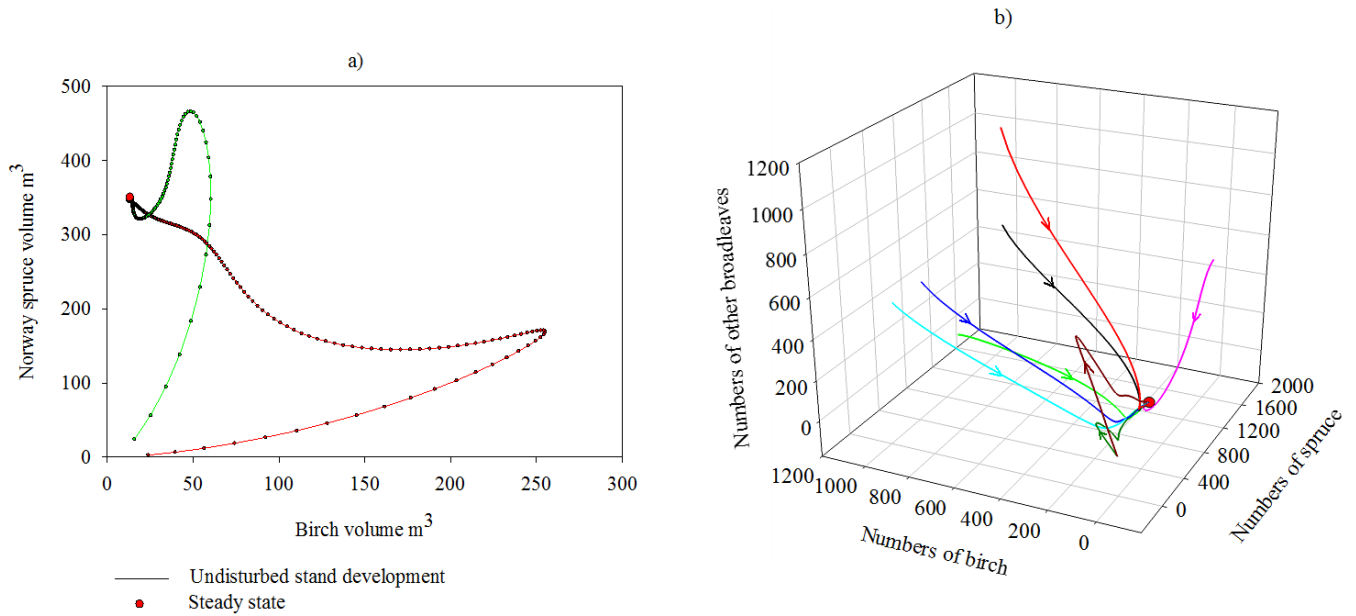

Figure 2, a,b. Stand development without harvest
    a) Spruce, birch mixture
    b) Four species, the development of Scots pine ($x_{20} = 100$) not shown

Based on this growth model and the data Figure 2a shows undisturbed volume development for a Norway spruce and birch mixture comparable with Figure 1. Figure 2a includes all four species. In all cases, the trajectories of undisturbed stands converge toward Norway spruce-dominated steady states, a prediction that makes perfect sense in the case of a boreal average-productivity site without natural and human disturbances (cf. Bollandsås 2008).

The tree diameters and diameter-specific pulp and sawtimber volumes are given in Table 2 and are based on Heinonen (1994). The per period gross revenues and variable harvesting costs are given as

$$R(\mathbf{h}) = \sum_{j=1}^{l} \sum_{s=1}^{n} \left(p_{1j}v_{1sj} + p_{2j}v_{2sj} \right) h_{sj}, \tag{27}$$

$$C_v(\mathbf{h},\mathbf{k}) =$$

$$\sum_{j=1}^{l} \gamma_{ju0}\gamma_{ju1} \sum_{s=1}^{n} h_{js} \left(\gamma_{ju2} + \gamma_{ju3}v_{js} + \gamma_{ju4}v_{js}^{2} \right) + \tag{28a}$$

$$\gamma_{u5} \sum_{j=1}^{l} \sum_{s=1}^{n} h_{js}v_{js} + \gamma_{u6} \left(\sum_{j=1}^{k} \sum_{s=1}^{n} h_{js}v_{js} \right)^{0.7} + \tag{28b}$$

$$\sum_{j=1}^{l} \sum_{s=1}^{n} k_{js} \left(\gamma_{u7} + \gamma_{u8}v_{js} \right), \ u = th, cl, \tag{28c}$$

where p_{1j} and p_{2j} are sawtimber and pulpwood prices, v_{1sj} and v_{2sj} sawtimber and pulpwood volumes per tree, v_{js} is the total tree volume, and γ_{juk}, $j = 1,...,l, u = th, cl, k = 1,...,9$ are parameters (Appendix 1, Table 3). This specification is based on detained empirical logging experiments by Nurminen et al (2006), and it includes cutting (28a), hauling (28b), and felling (28c) costs separately. The cutting cost per tree is higher for thinning compared to clearcut ($\gamma_{jth1} > \gamma_{jcl1}, j = 1,...,l$). Variable harvesting costs increase with total harvested volume but decrease with tree volume. In computing the hauling cost (28b) there is no need to separate the tree species. The prices for sawtimber and pulpwood are given in Appendix 1, Table 4. Fixed cost C^{f} equals €500 and refers to moving the harvester to the site.

Biodiversity is measured with the Simpson (1949) (or Herfindahl 1950) diversity index. Site diversity may depend on both on species and tree sizes (Eggers 2018, Duncker 2012, O'Hara 2014), and we take each size class and each tree species as "species". Thus, trees in each size class are different "species" and the maximum number of species equals $l \times n$. The Simpson index is defined as

$$d\left(\mathbf{x}\right)=1-\frac{\sum\limits_{j=1}^{l}\sum\limits_{s=1}^{n}x_{js}\left(x_{js}-1\right)}{\sum\limits_{j=1}^{l}\sum\limits_{s=1}^{n}x_{is}\left(\sum\limits_{j=1}^{l}\sum\limits_{s=1}^{n}x_{js}-1\right)},\ d\left(\mathbf{x}\right)\in\left[0,1\right].\tag{29}$$

The value of Simpson index is high when the stand carrying capacity is evenly allocated across tree species and size classes. Because of the lack of empirical data, it is assumed that the value ES depend linearly on the diversity measure, i.e. $\tilde{A}\{E[d(x)]\}=Ad(x)$, where A is a constant. The value of A will be varied widely to reveal the effects of ES valuation on wood production and cuttings.

Optimization method and algorithms

The optimization problem (18)–(29) is a dynamic discrete-time problem. Complications arise from nonlinearities, potential nonconvexities, many state variables (44–48), and the mixed-integer feature of the optimized variables. We search for optimal solutions by applying a tri-level computational structure. At the upper level, we optimize the rotation period; at the middle level, the timing and number of thinnings; and at the lowest level (given the rotation length and timing of thinnings), the number of trees harvested from each size classes and species. As the number of trees is considered a continuous variable, the lowest level problem is solved using gradient-based methods and AMPL/Knitro optimization software (version 10.2), which enables the use of four state-of-the-art interior-point and active-set methods. The 44–48 state variable problems with 80–180 time periods can be computed within a couple of seconds. Potential nonconvexities are handled with a multi-start procedure. The middle level problem is to optimize the 0-1 binary variables for thinning timing. This is performed using hill climbing and genetic algorithms. In addition, the functioning of these algorithms is spot-checked by computing the outcomes of all conceivable timing combinations. To find the optimal rotation length, the middle and lower level optimization is repeated for rotation periods between 80 and 180 years by applying a five-

year period length. Parallel computation is utilized whenever possible. When 180-years rotation yields the highest bare land value, we additionally compute an approximation for the optimal infinite horizon solution. This computation includes up to seven optimized harvests (both timing and number of trees harvested) before reaching a steady state harvesting cycle with an optimized interval length between the harvests. Using an Intel (R) Xeon (R) E5-2643 v3 @3.40GHZ, 24 logical processor computer, solving the infinite horizon approximation takes 50–120 hours. More details for these procedures are explained for the case of single-tree species model in Sinha et al. (2017).

5 Results

The Faustmann-Hartman model with varying number of tree species

Figure 3a shows the volume developments of unharvested (average-fertility sites) and that adding tree species has a surprisingly small effect on the total stand volume, which reaches a maximum of $500m^3$ at the age of 100 years. Figure 3b reveals that the mixed stand is dominated by Norway spruce. In Table 1, the maximized bare land value increases with the number of commercial species while adding noncommercial other broadleaves decreases the bare land value. Optimal rotation varies between 60 and 50 years and the rotation for single-species Norway spruce is in line with earlier studies (Niinimäki et al 2012). Annual yield and the discounted level of ES increases with the number of tree species.

The diversity of unharvested stands reach a maximum at stand ages between 100 and 110 years (Figure 3c), because the number of trees in various size classes is highest at these ages and at greater ages the stand becomes dominated by Norway spruce. Including ES values (mixed stand with all four species) lengthens rotation to 110 years (Figure 3d). When $A = 6460$, this rotation is locally optimal simultaneously with abandoning the clearcut. Given $A > 6460$, the latter becomes globally optimal. This value of parameter A will be used as a benchmark in analyzing the effects of ES preferences for the generalized model with thinning.
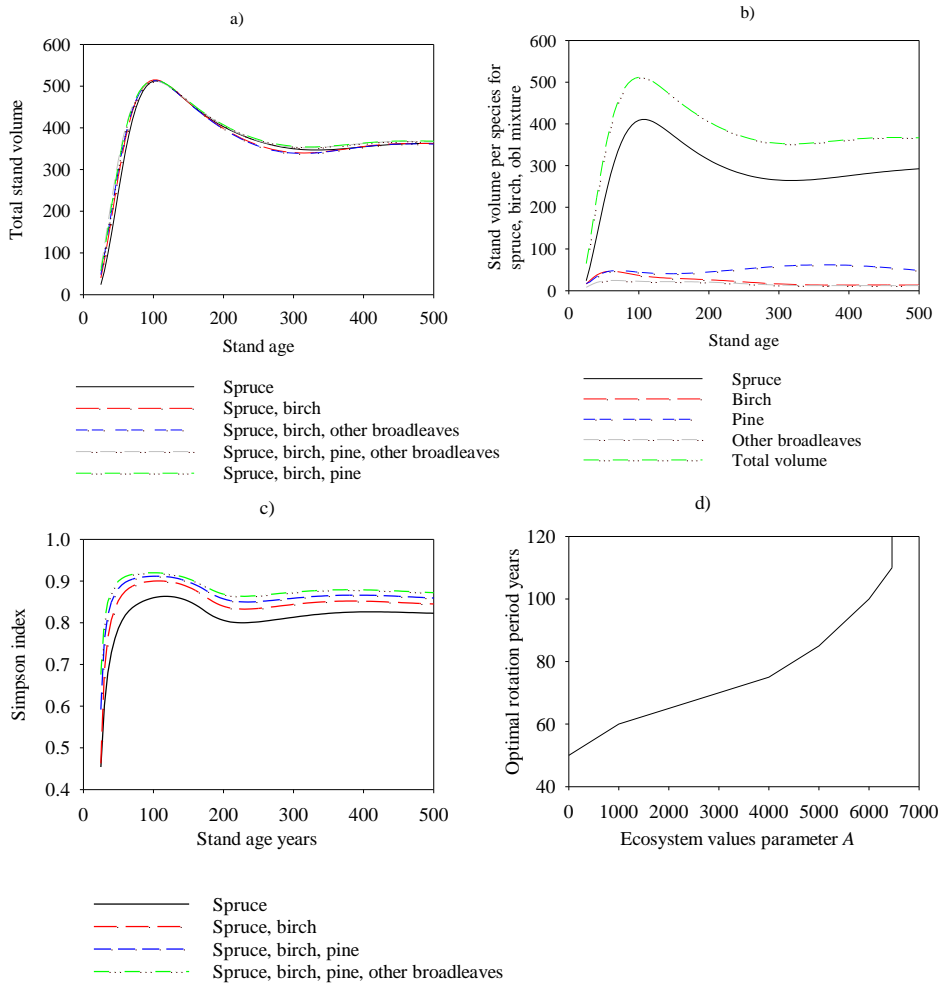
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Figure 3a-d. Stand development and rotation within the Faustmann-Hartman model
Note: a) and c) different mixtures, b) and d) mixed stands with spruce, birch, pine other broadleaves, d) interest rate 3%, w=0, initial state see Table 1.

Table 1. Optimal solutions within the Faustmann-Hartman model

Species mixture	Bare land value € ha^{-1}		Rotation years		Annual yield m^3 ha^{-1}		Discounted ES with $A=1$	
	w=0	w=1500	w=0	w=1500	w=0	w=1500	w=0	w=1500
Spruce	3377	1513	55	60	6.2	6.4	1.22	1.36
Spruce, birch	3407	1540	55	55	6.8	6.8	1.79	1.79
Spruce, birch, pine	3650	1767	50	55	7.1	7.1	1.78	1.96
Spruce, birch, pine, other bl	3426	1551	50	55	6.8*	6.8*	1.89	2.07

Note: w=regeneration cost (€), ES=ecosystem services bl=broadleaves, interest rate 3%, initial state at $t_0 = 20$: spruce 1750, birch 1000, pine 500, other broadleaves 500, *does not include other broadleaves

Stand management with optimized thinning and rotation but without preferences for ecosystem services

Figures 4a, b show bare land values as functions of rotation lengths given optimized thinning timing and the harvested number of trees from different size classes and species. Assuming no regeneration cost, the rotation periods maximizing the bare land values are finite and vary between 100 and 120 years, i.e. are approximately twice as long as without thinning. The rotation periods become infinitely long under a positive regeneration cost (Figure 4b) of €1500, implying the optimality of continuous cover forestry.

Comparing Tables 1 and 2 shows that including thinning increases the bare land values c.a. 20% when regeneration cost are zero, and c.a. 70% with a positive regeneration cost and when the inclusion of thinning causes a switch from rotation forestry to continuous cover forestry. Note that average annual wood output decreases ca. 20% simultaneously when the optimized thinning and continuous cover forestry cause the 70% increase in bare land value, thus demonstrating the misleading nature of wood output as a guiding objective in forestry. Comparing the present value of ES between the solutions for the Faustmann-Hartman model and the model with thinning shows that both thinning with longer rotations and the continuous cover solution increases stand diversity and ES without exceptions.

Figures 4c and d compare the rotation and continuous cover solutions. They are both dominated by Norway spruce albeit the fraction of birch is ca. 24% in the continuous cover steady-state solution (with an optimal 15-year harvesting period). The fraction of pine is kept negligible because of low natural regeneration in a relatively dense spruce dominated stand. Other noncommercial other broadleaves are felled (but left to the site) at the harvesting dates in both type of solutions. If noncommercial broadleaves are left growing in the continuous cover solution, our computation shows that they will take over the stand and the fraction of valuable species and their harvesting decreases toward zero.[6]

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[6] The outcome is called "high grading", i.e. "take the best, leave the rest". We note that high grading is not economically valid argument against continuous cover forestry albeit it may occur in open access situations.
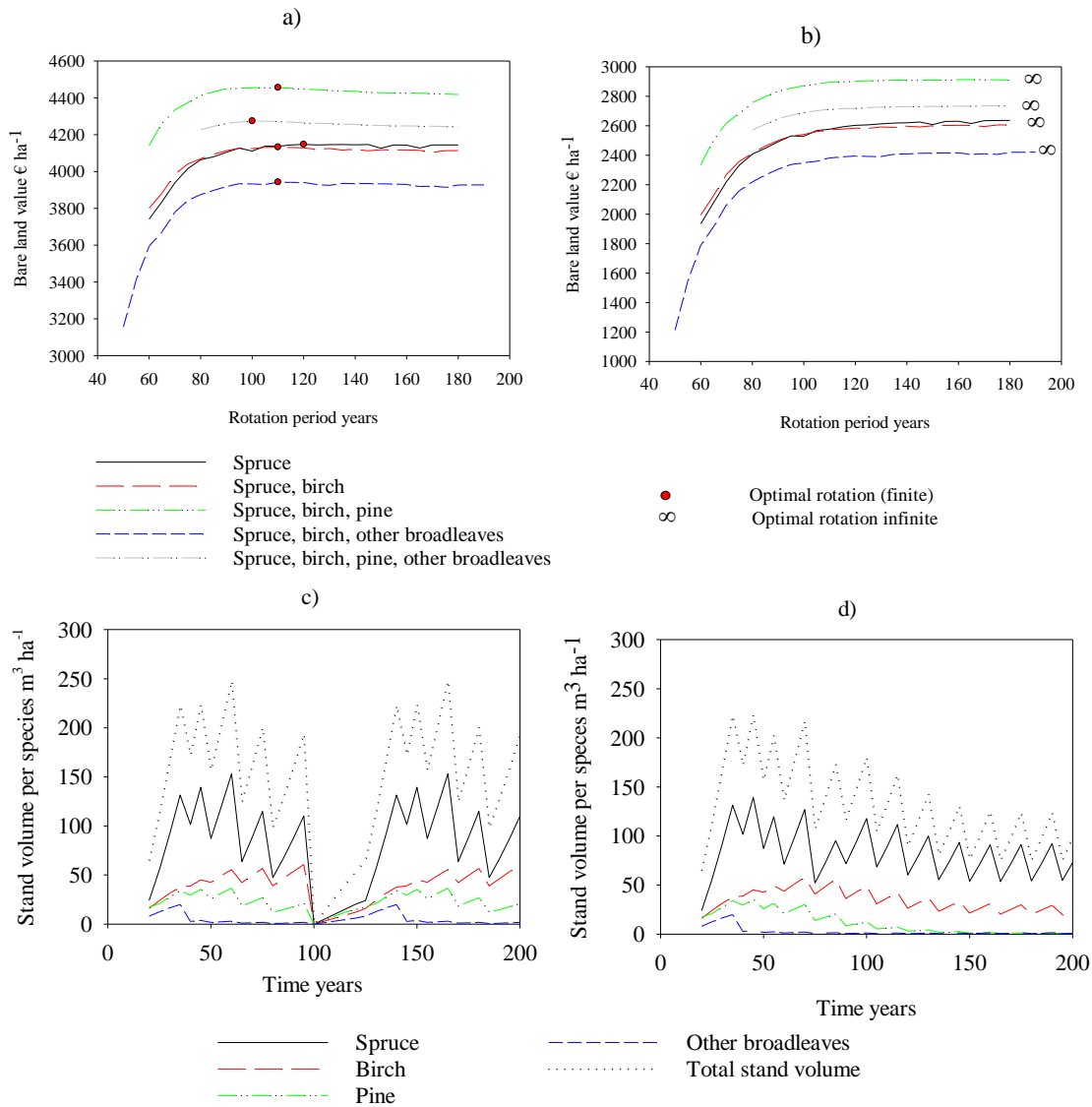
Figure 4a–d. Rotation vs continuous cover forestry without ecosystem services
a), c) Regeneration cost zero, a), c) Regeneration cost €1500, b), d): Interest rate 3%

Table 2. Optimal solutions for harvesting net revenues maximization with thinning

Species mixture	Bare land value € ha⁻¹		Rotation years		Annual yield m³ ha⁻¹		Discounted ES with $A=1$	
	w=0	w=1500	w=0	w=1500	w=0	w=1500	w=0	w=1500
Spruce	4148	2644	120	∞	6.4	4.7[**]	1.90	2.00
Spruce, birch	4132	2610	110	∞	7	5.2[**]	2.50	2.65
Spruce, birch, pine	4457	2917	110	∞	7.2	5.1[**]	2.67	2.82
Spruce, birch, pine, other bl	4274	2739	100	∞	7.2[*]	5.1[**)*]	2.72	2.91

Note: w= regeneration cost, ES=ecosystem services, interest rate 3%, initial state as in Table 1.
[*]Does not include other broadleaves, [**]Steady state yield.

Stand management with optimized thinning and preferences for ecosystem services

Including preferences for ES lengthens optimal rotation (Figure 5a,b) as in the model without thinning. With optimized thinning an ecosystem preference parameter equal to ca. $A = 600$ is enough to imply infinite rotation and a switch to continuous cover forestry. Note that within the Faustmann-Hartman model the same level of ES valuation produces 57-year rotation (Figure 3d) and the rotation does not become infinitely long until $A = 6460$.
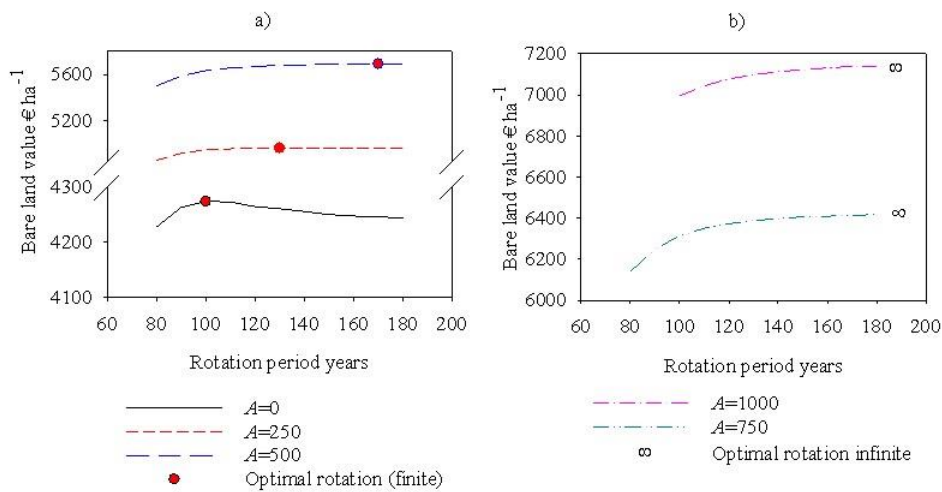

Figure 5a,b. The effects of ecosystem services on optimal rotation
Note: regeneration cost zero, interest rate 3%, initial state as in Table 1.

Comparing Figures 6a and 4d shows that ES increase overall stand density from ca. $100 m^3 ha^{-1}$ to $116 m^3 ha^{-1}$ and the steady-state harvesting interval from 15 to 20 years. Additionally, the standing volumes of birch and noncommercial other broadleaves increase, while the relative volume of Norway spruce decreases from 0.7 to 0.56. Figure 6b shows the development of the main economic variables over time. Harvesting costs are ca. 20% of the harvesting revenues. A drop occurs in the Simpson index at the dates of harvesting, as harvesting decreases the number of large size classes for each species. This "thinning from above" can be seen from Figures 6c-f along with how including ES changes the stand-

steady state size structure and harvest of each species. Including ES decreases the number of trees in small size classes, while the number of trees in large size classes increases. Postponing tree harvest to larger trees becomes optimal for all tree species. This is most clear for noncommercial other broadleaves.
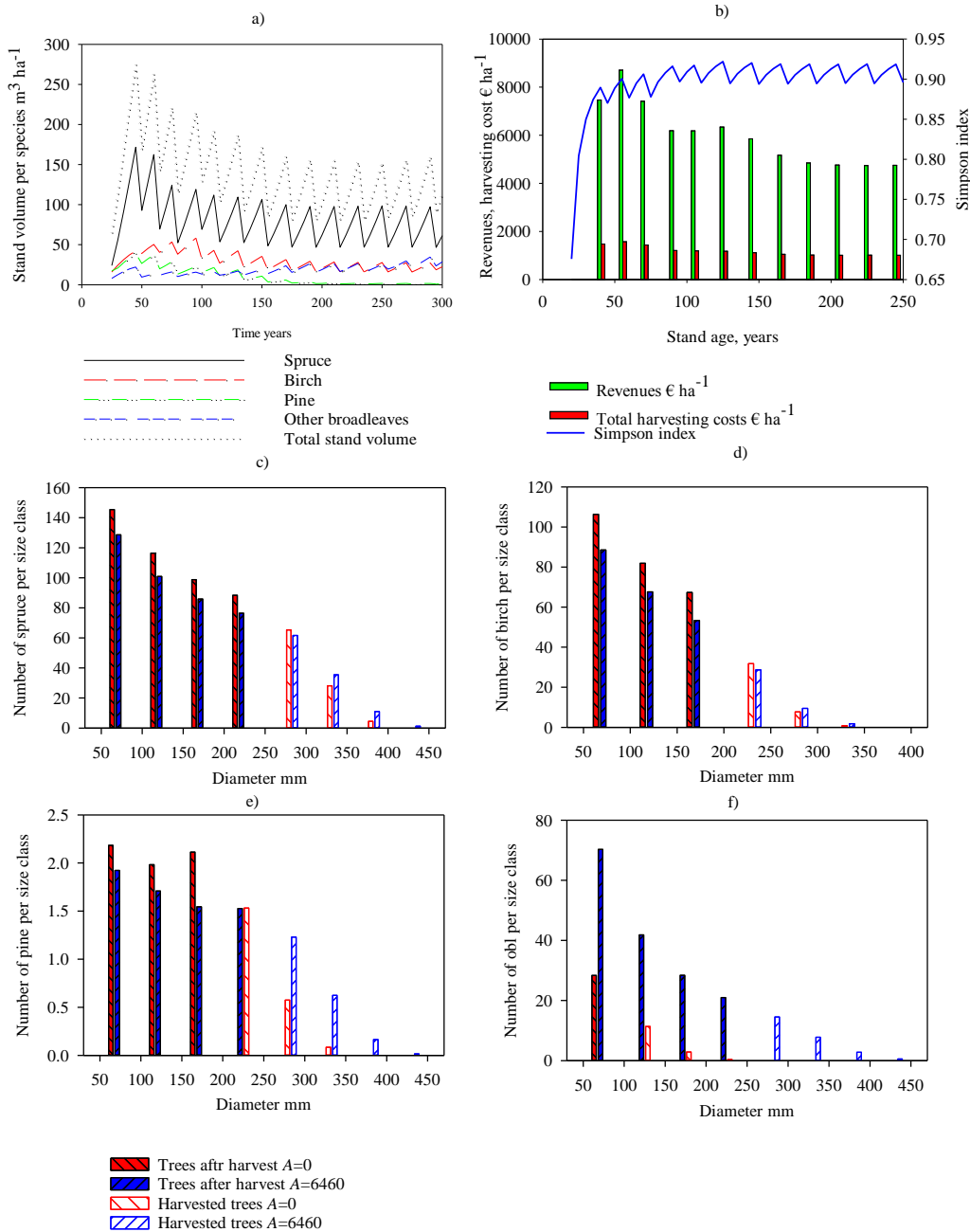

Figure 6a-f. Optimal solution with preferences for ecosystem services
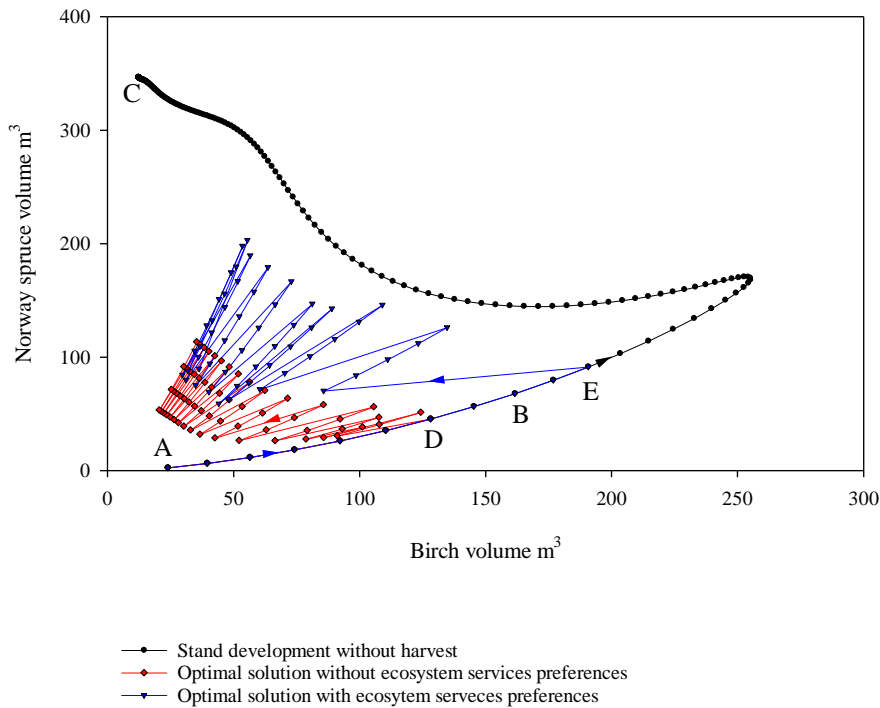Note: Interest rate 3%, all four tree species

Figure 7. Solutions for a Norway spruce birch mixture.
Note: Interest rate 3%, *A=0* or *A=6460*

Another perspective to the effects of valuing ES can be obtained by depicting optimal solutions for a two-species mixture (cf. Figures 1 and 2a). Given the Faustmann-Hartman model and no ES preferences the optimal rotation period equals 55 years and the solution cycles from bare land to point *B* via *A* and back to bare land. If ES preferences are high enough, clearcut is abandoned in the Faustmann-Hartman model and an unharvested stand develops toward the steady state at point *C*. In contrast with optimized thinning clearcut is abandoned both with and without ES preferences. In the former case, the solution proceeds without harvest to state *D* and then converges toward a 20-year continuous cover cycle. With ES preferences the solution proceeds further without harvest (state *E*) and converges toward a 30-year continuous cover cycle, where the average volumes of both species are higher.

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Increasing the value of ES decreases the net revenues from harvesting. In Figure 8a these costs are much higher in the Faustmann-Hartman model, and the cost of reaching the maximum level of ES is only ca. 95% lower when thinning and the continuous cover solution are applied. This is partly explained by the fact that continuous cover harvesting allows maintaining higher (species) diversity compared to the no-harvesting solution, where the stand develops toward a Norway spruce-dominated state (Figure 8b).
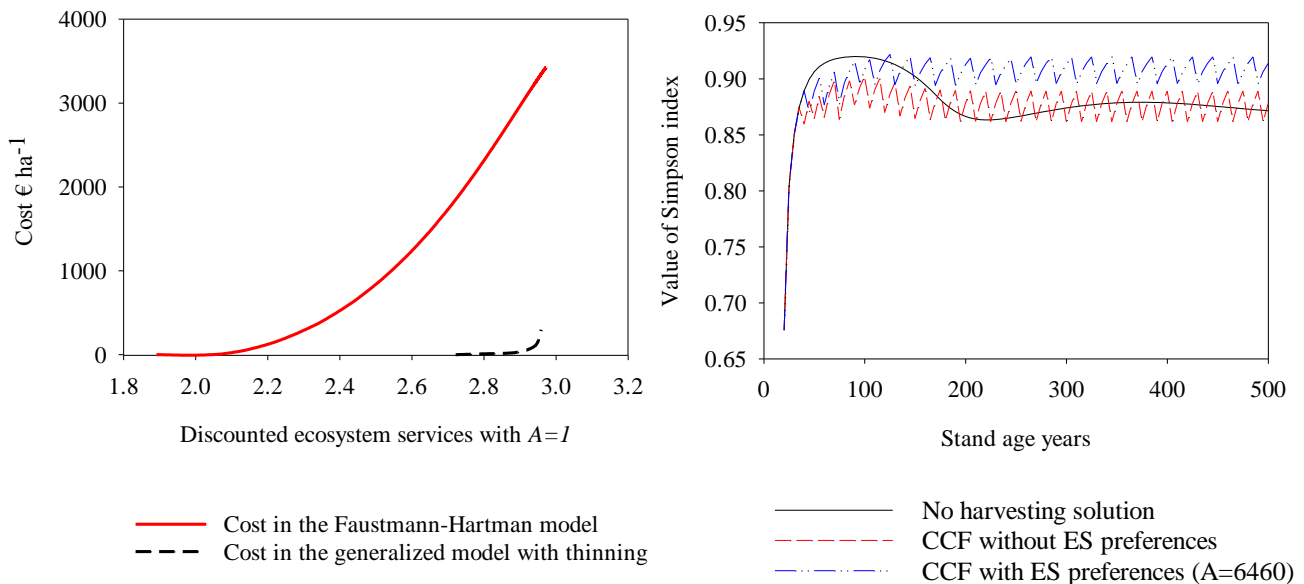
Figure 8a,b. Costs and production of ecosystem services
        Notes: Interest rate 3%, regeneration cost 1500€, all four tree species included

6 Conclusions

Our study shows that while it is important to proceed beyond the widely cited Faustmann-Hartman setup to richer descriptions of forests amenity values and ecosystem services, it is equally important to expand the set of forest harvesting activities from clearcutting to various forms of partial harvesting or thinning. Both theoretical and empirically detailed models show that by restricting economic analysis to optimizing the rotation period only is overly limited especially in the presence of heterogeneous mixed species

forests and preferences for ecosystem services. A wider set of management alternatives allows to adjust the mixture of naturally regenerating tree species during the rotation as well as maintaining harvest revenues albeit abandoning clearcuts. As a consequence both net revenues from harvesting and the value of ecosystem services tend to increase. Increasing the level of ecosystem services and stand diversity from the outcome based on wood production decreases revenues but this cost is much lower under extended management alternatives compared to the Faustmann-Hartman setup.

Several features of our model are very different compared to earlier-mixed species forest economic studies, making the comparison of their results difficult. For example, no earlier study presents results for mixed-species stands and optimization between rotation forestry with clearcuts and continuous cover forestry. Additionally, optimization results in earlier studies are not based on optimizing harvest timing in mixed-species continuous cover forests. Earlier models with ecosystem values have not included the management of naturally regenerating noncommercial species that may have ES value.

Our results can be compared to certain earlier views on the management of mixed-species forests. Filyushkina et al. (2018) write that while intensive management, such as clearcuts or frequent thinning, decreases forest ecosystem value, certain management may be beneficial. Our results are clearly in line with this view or more generally with the widely studied "intermediate disturbance hypothesis" stating that highest species diversity occurs in species communities with medium scale disturbances (Connell 1978). Using a large set of global data, Liang et al. (2016) show that tree species richness increases wood production. Additionally, they emphasize the re-evaluation of forest management strategies and the potential benefits from the transition of monocultures to mixed-species stands. These views obtain support from our results (Tables 1 and 2), but depending on whether the higher number of species is valuable either commercially or as sources of ES. Finally, we note that coming economic studies should still proceed in generalizing the description of forest amenity and ES values and in re-evaluating forest

management alternatives using various ecological models for mixed-species forests estimated for various

forest environments.

Appendix 1. Parameters for regeneration, transition and mortality functions

Table A1. Parameters for equations (24)–(26) (Bollandsås (2008).

	Norway spruce	Scots pine	Birch	Other broadleaves
ϕ_1	43.142	67.152	64.943	3.438
ϕ_2	0.051	0	0.104	0.193
ϕ_3	0.368	0	0.143	0.442
ϕ_4	0.741	1.205	1.205	1.205
ϕ_5	-0.157	-0.076	-0.161	0.170
ϕ_6	-2.291	-3.552	-0.904	-3.438
ϕ_7	0.018	-0.062	-0.037	-0.029
ϕ_8	0.066	0	0	0.123
ϕ_9	0.019	0.08	0.016	0.048
α_1	0.02	0.02	0.02	0.02
α_2	17.839	25.543	11.808	2.204
α_3	0.0476	0.0251		0.063
α_4	-11.585×10^{-5}	-5.660×10^{-5}	9.616×10^{-5}	-8.320×10^{-5}
α_5	0	0	-9.585×10^{-8}	0
α_6	-0.3412	-0.216	0	0
α_7	-0.024	-0.123	-0.152	-0.177
α_8	0.906	0.698	0.519	0.359
α_9	-0.268	-0.336	-0.161	0
μ_1	-2.492	-1.808	2.188	-1.551
μ_2	-0.020	-0.027	0.016	-0.011
μ_3	3.200×10^{-5}	3.300×10^{-5}	2.700×10^{-5}	1.400×10^{-5}
μ_4	0.031	0.055	0.030	0.016

Note: We set the latitude parameter equal to $L = 61.9$ and assume an average fertility site (S_{15}) where the height of 100 dominant trees equal 15m.

Table A2. Sawlog and pulpwood volumes $v_{1js}, v_{2js}, j=1,...,l, s=1,...,n$ (m^3) per tree.

Size class	Diameter cm	Norway spruce		Scots pine		Birch and other broadleaves	
		Pulp	Saw log	Pulpwood	Saw log	Pulpwood	Saw log
1	7.5	0.01374	0	0.03458	0	0.01591	0
2	12.5	0.06664	0	0.06659	0	0.07464	0
3	17.5	0.1669	0	0.10166	0.09764	0.18005	0
4	22.5	0.0808	0.23419	0.03905	0.27034	0.07854	0.25137
5	27.5	0.06482	0.44578	0.03001	0.48515	0.06655	0.45137
6	32.5	0.05975	0.68392	0.02750	0.74205	0.05827	0.69732
7	37.5	0.04978	0.96304	0.02647	1.04106	0.04978	0.96304
8	42.5	0.05039	1.25313	0.02596	1.38216	0.04865	1.24859
9	47.5	0.04324	1.57421	0.02567	1.76537	0.04463	1.55035
10	52.5	0.03925	1.89981	0.02549	2.29067	0.03891	1.86531
11	57.5	0.03317	2.21442	0.02537	2.65807	0.03685	2.18117
12	62.5	0.03073	2.56544	0.02529	3.16758	0.03268	2.49693

Table A3. Prices for saw timber and pulpwood, p_{1i}, p_{2i}.

	Norway spruce	Scots pine	Birch	Other broadleaves
Saw timber	58.44	58.64	49.73	0
Pulpwood	34.07	30.51	30.50	0

Table A4: Parameter values for the harvesting cost functions

Species	u	γ_{ju0}	γ_{ju1}	γ_{ju2}	γ_{ju3}	γ_{ju4}	γ_{ju5}	γ_{ju6}	γ_{ju7}	γ_{ju8}
Norway spruce	th	2.100	1.150	0.412	0.758	-0.180	2.272	0.535	0.826	0.244
	cl	2.100	1.000	0.412	0.758	-0.180	1.376	0.393	0.6132	0.2982
Scots pine	th	2.100	1.150	0.547	0.196	0.308	2.272	0.535	0.826	0.244
	cl	2.100	1.000	0.532	0.196	0.308	1.376	0.393	0.6132	0.2982
Birch	th	2.100	1.150	0.420	0.797	0.174	2.272	0.535	0.826	0.244
	cl	2.100	1.000	0.430	0.756	0.174	1.376	0.393	0.6132	0.2982
Other broadleaves	th	2.100	1.150	0.342	0.101	0	2.272	0.535	0.826	0.244
	cl	2.100	1.000	0.342	0.101	0	1.376	0.393	0.6132	0.2982

Note: Symbols, see equation (7).

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